Components of the third cumulant in a two-way classification with n observations per cell

BU-97-M

May 1958

by Glenn Atkinson

α

$$\frac{(c-1)(c-2)}{c}\left\{K_3(\alpha)+\frac{1}{r^2}K_3(\gamma)+\frac{1}{r^2n^2}K_3(\epsilon)+\frac{3}{r}\operatorname{cov}(\alpha\bar{\sigma}^2_\gamma)+\frac{3}{rn}\operatorname{cov}(\alpha\bar{\sigma}^2_\epsilon)+\frac{3}{r^2n}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)\right\}$$

β

$$\frac{(r-1)(r-2)}{r}\left\{K_3(\beta)+\frac{1}{c^2}K_3(\gamma)+\frac{1}{c^2n^2}K_3(\epsilon)+\frac{3}{c}\operatorname{cov}(\beta\bar{\sigma}^2_\gamma)+\frac{3}{cn}\operatorname{cov}(\beta\bar{\sigma}^2_\epsilon)+\frac{3}{c^2n}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)\right\}$$

γ

$$\frac{(c-1)(c-2)(r-1)(r-2)}{cr}\left\{K_3(\gamma)+\frac{1}{n^2}K_3(\epsilon)+\frac{3}{n}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)\right\}$$

$\operatorname{Cov}(\alpha\beta\gamma)$

$$(c-1)(r-1)\left\{\operatorname{cov}(\alpha\beta\gamma)+\frac{1}{c}\operatorname{cov}(\alpha\bar{\sigma}^2_\gamma)+\frac{1}{r}\operatorname{cov}(\beta\bar{\sigma}^2_\gamma)+\frac{1}{cn}\operatorname{cov}(\alpha\bar{\sigma}^2_\epsilon)+\frac{1}{rn}\operatorname{cov}(\beta\bar{\sigma}^2_\epsilon)+\frac{2}{crn}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)+\frac{1}{cr}K_3(\gamma)+\frac{1}{crn^2}K_3(\epsilon)\right\}$$

$\operatorname{Cov}(\alpha\sigma^2_\gamma)$

$$(c-1)(r-1)\left\{\operatorname{cov}(\alpha\bar{\sigma}^2_\gamma)+\frac{1}{n}\operatorname{cov}(\alpha\bar{\sigma}^2_\epsilon)+\frac{1}{rn}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)+\frac{1}{r}K_3(\gamma)+\frac{1}{rn^2}K_3(\epsilon)\right\}$$

$\operatorname{Cov}(\beta\sigma^2_\gamma)$

$$(r-1)(c-1)\left\{\operatorname{cov}(\beta\bar{\sigma}_\gamma)+\frac{1}{n}\operatorname{cov}(\beta\bar{\sigma}^2_\epsilon)+\frac{1}{cn}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)+\frac{1}{c}K_3(\gamma)+\frac{1}{cn^2}K_3(\epsilon)\right\}$$

$\operatorname{Cov}(\alpha\sigma^2_\epsilon)$

$$r(c-1)(n-1)\left\{\operatorname{cov}(\alpha\bar{\sigma}^2_\epsilon)+\frac{1}{r}\operatorname{cov}(\gamma\bar{\sigma}^2_\epsilon)+\frac{1}{rn}K_3(\epsilon)\right\}$$

$\operatorname{Cov}(\beta\sigma^2_\epsilon)$

$$c(r-1)(n-1)\left\{\operatorname{cov}(\beta\bar{\sigma}^2_\epsilon)+\frac{1}{c}\operatorname{cov}(\gamma\sigma^2_\epsilon)+\frac{1}{cn}K_3(\epsilon)\right\}$$

$\mathrm{Cov}(\gamma\bar{\sigma}^2_\epsilon)$ $\quad (c-1)(r-1)(n-1)\left\{ \mathrm{cov}(\gamma\bar{\sigma}^2_\epsilon) + \frac{1}{n} K_3(\epsilon) \right\}$

ϵ $\quad \frac{cr(n-1)(n-2)}{n} \quad K_3(\epsilon)$

One observation per cell

α $\quad \frac{(c-1)(c-2)}{c}\left\{ K_3(\alpha) + \frac{1}{r^2} K_3(\gamma) + \frac{3}{r} \mathrm{cov}(\alpha\bar{\sigma}^2_\gamma) \right\}$

β $\quad \frac{(r-1)(r-2)}{r}\left\{ K_3(\beta) + \frac{1}{c^2} K_3(\gamma) + \frac{3}{c} \mathrm{cov}(\beta\bar{\sigma}^2_\gamma) \right\}$

$\mathrm{Cov}(\alpha\beta\gamma)$ $\quad (c-1)(r-1)\left\{ \mathrm{cov}(\alpha\beta\gamma) + \frac{1}{c} \mathrm{cov}(\alpha\bar{\sigma}^2_\gamma) + \frac{1}{r} \mathrm{cov}(\beta\bar{\sigma}^2_\gamma) + \frac{1}{cr} K_3(\gamma) \right\}$

$\mathrm{Cov}(\alpha\sigma^2_\gamma)$ $\quad (c-1)(r-1)\left\{ \mathrm{cov}(\alpha\bar{\sigma}^2_\gamma) + \frac{1}{r} K_3(\gamma) \right\}$

$\mathrm{Cov}(\beta\sigma^2_\gamma)$ $\quad (r-1)(c-1)\left\{ \mathrm{cov}(\beta\bar{\sigma}^2_\gamma) + \frac{1}{c} K_3(\gamma) \right\}$

γ $\quad \frac{(c-1)(c-2)(r-1)(r-2)}{cr} \quad K_3(\gamma)$